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A SUMMARY OF SEDIMENT TRANSPORTATION MECHANICS

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Introduction

The development of the mechanics of transportation of sediment by fluids has been carried on mainly in connection with the control and development of streams where its greatest economic importance lies. The difficulties of controlling rivers are concerned largely with such matters as the ability of the flow to erode the banks and bed or the possibility of aggradation due to insufficient transporting capacity. Thus, stream control problems are to a great extent problems in sediment transportation. Although sediment transportation is closely identified with streams, its application is by no means limited to this field. It is of importance in many industrial processes where solids are transported by liquids or gases or where solids need to be mixed and/or separated from fluids, and in the important field of transportation of soil by wind.

In this paper a brief outline will be given of the mechanics underlying this subject. No attempt will be made to discuss the many important practical problems which provided the incentive for developing this special branch of fluid mechanics.

Description of Transportation Mechanism

Consider a smooth bed of sand in a flume with water flowing over it at a velocity low enough so that no sand is moved. Then let us observe the bed as the flow rate and velocity are increased. The

first motion will be a rolling or sliding of the individual grains, and will be intermittent and occur only in isolated spots of the bed, indicating a random fluctuation in time and space of the forces on the particles. Individual grains will be rotated up off the bed about their downstream edge as if they were about to be rolled, and may oscillate several times before settling back to the bed or actually being set into motion. Some grains slide along without rolling and others move with a combined sliding and rolling motion. All motion at this incipient stage occurs in short steps and is usually intermittent.

As the flow rate is increased, the frequency of the steps of individual grains increases and some grains are actually lifted off the bed and execute short trajectories or saltations as they are carried downstream. As the flow is increased still more, the saltations increase in length and height and some of the grains are lifted by the cross components of turbulence and may execute extended excursions into the flow. Such particles are said to be suspended. At more advanced rates of transportation when much sediment is in suspension the material is seen to move in billowing streaks of clouds much like the dust streaks shown in Fig. 1. These streaks, like the particle motions described above, are not steady, but form at random in various parts of the bed, persist for varying periods of time, and then their formation ceases and the clouds thus formed seem to lose their identity in the random suspension existing in the flow.

If a given flow that produces motion at the bed is allowed to persist for a time, the familiar ripple marks or dunes will form on the bed, as illustrated in Fig. 2b. The dunes move downstream at low velocities compared to the water velocity as the sand is carried up the flat slope and deposited on the steep downstream face, much as occurs in the well known aeolian sand dunes found in deserts. If the flow rate is now increased, a condition will be reached where the dunes are flattened out and the bed becomes smooth again (Fig. 2c). At still higher

velocities and transportation rates, dunes will form again, but unlike those described above, they move upstream instead of downstream (Fig. 2d). This motion is attained because the downstream face of the dunes erodes and the material is deposited on the upstream faces. Gilbert (1) has referred to such sand waves as antidunes. Antidunes are accompanied by waves on the water surface immediately above the dune area. These waves form intermittently at various places on the stream surface and travel upstream, getting steeper with time and finally breaking and disappearing.

Fig. 3 is a view looking upstream at dunes formed in a laboratory flume. Dunes or ripple marks are also formed by wind erosion and wave motion, and they have even been observed in pipes carrying sediment-laden flow.

Initiation of Movement

The conditions in a stream at which motion of the sediment in the bed just begins is of considerable practical interest, and has been the subject of a number of investigations. To investigate this condition, let us analyze the forces acting on a single grain resting on a bed of loose sediment. Although natural sediment grains are never spherical, we can express their size by an equivalent diameter, D , which we shall not define rigorously at this time.

The forces acting on a grain of sediment lying in a horizontal bed of similar grains over which a fluid flow is occurring, are the weight or gravity force and the hydrodynamic lift acting in the vertical direction, and the friction and hydrodynamic drag acting horizontally in the direction of flow. The lift is probably less important than the drag, and as a matter of fact, the existence of lift is sometimes questioned. However, both analytical (2) and experimental (3), (4) studies seem to establish its presence. In setting up the equilibrium of a grain on a bed, the lift has not been considered explicitly. But since the lift and drag are both approximately proportional to the velocity squared, the

analysis does in effect consider the lift.

As shown in Fig. 4, taken from the work of C. M. White (5), a grain in the bed of a flowing stream may be considered as being acted upon by a fluid force in the flow direction of $\tau_o D^2/\eta$ and a gravity force

$(\rho_1 - \rho) g \frac{\pi}{6} D^3$ where τ_o is the fluid shear stress at the bed, η is a packing coefficient such that D^2/η is the average bed area occupied by each grain, ρ_1 and ρ are the densities of the sediment and fluid, respectively, D is the diameter of the grain, and g is the acceleration of gravity. Taking moments about the point of support and rearranging the terms, we get

$$\tau_c = \eta \frac{\pi}{6} (\rho_1 - \rho) g D \tan \theta \quad (1)$$

where θ is the angle of repose of the sediment in the fluid and τ_c is the shear that will just start motion.

White found from experiments with a sand bed in a nozzle that Eq. 1 fitted the observed results. His nozzle was designed to give uniform shear stress at the bed although the average velocity was increasing as the flow passed through the nozzle. By using oil as well as water, he was able to get a wide variation in D/δ , the ratio of sand size D to thickness of the laminar sublayer δ where $\delta = 11.6 \nu / \sqrt{\tau_o/\rho}$ and ν is the kinematic viscosity of the fluid. For values of D/δ less than about 0.2 when the sand grain is completely enveloped in laminar flow, the shear required to start motion is greater than in cases where the laminar layer is thin and the flow around the grain is turbulent. This difference was ascribed to the turbulent fluctuations in velocity which at the bed may amount to as much as 50 per cent of the local mean velocity. Since the shear varies as the velocity squared, the maximum value of the shear exceeds twice the average value. For rounded sand in steady viscous flow, White obtained the following relation for initiation of sediment movement,

$$\tau_c = 0.18 (\rho_1 - \rho) g D \tan \theta \quad (2)$$

He estimated that for turbulent flow in his apparatus, τ_c would be reduced about 40 per cent and that in streams with fully developed turbulence the reduction would be greater still. Introducing into Eq. 2 values of the density for water and quartz sand, a value for $\tan \theta$ of about unity, and making the indicated 40 per cent reduction, we get

$$\tau_c = 12 D \quad (3)$$

where D is measured in feet and τ_c is in pounds per square foot.

Rubey (6) made an analysis of the initiation of motion similar to White's but he expressed drag, F , on the grain by the familiar impact formula

$$F = C_1 D^2 \rho \frac{u_o^2}{2}$$

where C_1 is a coefficient and u_o is the velocity experienced by the grain. He concluded that his analysis could predict the behavior of coarse grains but that the formulas gave values of u_o that were much lower than actually needed to move fine materials. Rubey pointed out that τ_c is proportional to u_o^2 and so that his conclusions are substantially in agreement with those of White.

If we substitute into Eq. 3 the relation $\tau_c \sim u_o^2$ and then cube both sides of the equation. we get the result $D^3 \sim u_o^6$ which is the well known sixth power law used by geologists and engineers since last century. In words it states that the weight or volume of the largest particles that can be moved by a stream varies as the sixth power of the velocity. It

must be pointed out that u_0 is the velocity in the neighborhood of the grain and not the average stream velocity since this error is made commonly.

Many observations of the shear, τ_c , to start motion of sediment have been made in Laboratory flumes. The results vary considerably but do not appear to disagree with those presented above (7).

Shields (8) set up the problem of initiating motion using the same equations as Rubey (6). He then obtained the velocity, u_0 , at the grain from the von Karman-Nikuradse velocity distribution law and obtained the relation,

$$\frac{\tau_c}{(\rho_1 - \rho) g D} = f\left(\frac{u_* D}{\nu}\right) = f_1\left(\frac{D}{\delta}\right) \quad (4)$$

where $u_* = \sqrt{\tau_0/\rho}$.

Data obtained by Shields from experiments with uniform size materials with a large range of specific weights are shown in Fig. 5, plotted according to Eq. 4. These results show a tendency for τ_c to reach a minimum in the neighborhood of $u_* D/\nu = 10$. White also found that the apparent τ_c is smaller for values of the parameter $u_* D/\nu$ in excess of 3.5 where the boundary is essentially hydrodynamically rough, than it is for values of the parameter of less than 2. However, White did not predict that τ_c would increase for higher values of the parameter. The discrepancy between these two sets of measurements probably lies in the different methods of defining τ_c , and in the different flow systems used. White determined the point of initiation of transportation visually and his flow was accelerating through the measuring section so that the turbulence was not fully developed in the flow cross section. Shields, on the other hand, worked in flumes with uniform

flow and fully developed velocity profiles. Also in Shields' work the value of τ_0 for beginning transportation is defined as the value of τ_0 in an empirical equation of rate of transportation as a function of τ_0 at which the transportation rate goes to zero. The value τ_c is therefore obtained by extrapolation of an empirical curve.

From Fig. 5 it will be noted that for the highest value of the parameter $u_* D / \nu$, the ordinate seems to be approaching a constant value of about 0.06, or

$$\frac{\tau_c}{(\rho_1 - \rho) g D} = 0.06$$

If we introduce values of ρ_1 and ρ for quartz sand (specific gravity 2.65) and water, respectively, into this equation, we get for the foot-pound system

$$\tau_c = 6.2 D \quad (5)$$

This value is about half that obtained by White (Eq. 3) and since White anticipated lower values for flows with fully developed turbulence, the discrepancy is not unreasonable.

As shown in Fig. 5, Shields obtained a remarkable correlation between $u_* D / \nu$ and the shape of dunes

formed on the bed. Since very little work has been done on the subject of bed dunes, there are no data with which to compare these results. Dunes, or ripple marks, as they are usually referred to in the geologic literature, are observed commonly in streams, in wind-blown sediments, and in the bottom of oceans and lakes where their occurrence has been observed (19) in water depths as great as 498 ft. Their shapes vary from those that are even more regular than shown in Fig. 3 to a series of individual bars reminiscent of scales on a fish and actually referred to by Shields in Fig. 5 as "scales".

Two attempts to analyze dune formation have come to the attention of the author. von Karman (10) reasoned that the transportation rate, and hence the local velocity, should be uniform along the dune profile and set up his equations to satisfy this relation. He obtained an expression for the wave length in terms of the surface velocity and shear. Anderson (11) reasoned that in a stream, surface waves induced periodic oscillations in velocity near the bed that resulted in variable transportation rates along the bed with the result that bed undulation formed. He obtained an expression for d/λ , the ratio of flow depth d , to dune wave length λ , as a function of Froude number $F = u/\sqrt{gd}$ which indicates that d/λ decreases as F increases. Anderson's results cannot be applied to wind transportation or to oceans or lakes without some modification since the ordinary Froude number has no significance in these cases.

Bed-Load Transportation

The term, bed-load, as used in the following discussion is defined loosely as the sediment that is transported on or near the bed. The rate of bed-load transportation is usually measured in weight per unit time per unit width of stream.

The first expression for the transportation of sediment to appear in the literature was developed by Du Boys (12) and is

$$q_s = B \tau_o (\tau_o - \tau_c) \quad (6)$$

where q_s is the transportation rate in weight per unit width and time, B is a coefficient, τ_o is the fluid shear at the bed, and τ_c is the critical value of τ_o for which transportation begins. Many experiments have been made to determine the quantities B and τ_c and values of these factors (13) are tabulated in the literature. In the course of experiments,

workers have developed empirical formulas to fit their data that deviate somewhat from the DuBoys formula, but most of these still retain the idea that motion begins when some critical value of some hydraulic quantity, such as shear or discharge, is reached. Johnson (14) has compared several of these formulas and concluded that they fit the data equally well.

Einstein (15) was the first to break away from the idea of a critical quantity at incipient transportation and adopted instead a probability concept. He first introduced the idea that grains move in steps or jumps whose length, L , are proportional to the particle diameter, D . He then argues that the number of grains per second that pass a given cross section of unit width is equal to the number of grains in the surface layer of the area of unit width and length L , times the probability that the local drag or lift will be great enough to set a particle in motion. With the aid of these concepts and dimensional analysis, he deduces the transport equation,

$$\phi = f(\psi) \quad (7)$$

where

$$\phi = \frac{1}{F} \frac{q_s}{\rho_1 g} \sqrt{\frac{\rho}{\rho_1 - \rho}} \frac{1}{D \sqrt{g D}}, \quad (8)$$

$$\psi = \frac{\rho_1 - \rho}{\rho} \frac{D}{RS} \quad (9)$$

and (16)

$$F = \sqrt{\frac{2}{3} + \frac{36\mu^2}{gD^3\rho(\rho_1 - \rho)}} - \sqrt{\frac{36\mu^2}{gD^3(\rho_1 - \rho)}} \quad (10)$$

In these equations R is the hydraulic radius, S is the slope of the stream, μ is the absolute viscosity of the fluid, and the other quantities are as defined previously. The dimensionless quantity ϕ , is

seen to be directly proportional to q_s , the transportation rate. Since $R S$ in Eq. 9 is proportional to τ_0 , the dimensionless quantity, ψ , is inversely proportional to τ_0 .

Fig. 6 shows data for uniform size sands from the work of Gilbert (1) and of Einstein plotted according to the $\phi - \psi$ coordinates of Eq. 7. Despite the scatter of the points there is no doubt that the formula describes the experimental results reasonably well, especially for low rates of transportation. The curves on Fig. 6 marked "1" and "2" are theoretical curves fitted to the data. Similar data for graded sands showed so much scatter on a graph like that of Fig. 6 that it was not possible to represent them on the $\phi - \psi$ coordinates. This disparity in behavior between sorted and graded sands has not been explained. On the other hand, bed load measurements by Einstein on two small streams (17) where the sediment size varied over a wide range fitted the curves of Fig. 6.

Because ψ contains the reciprocal of the bed shear τ_0 , the conditions for large values of τ_0 are all compressed in the small region near $\psi = 0$. If the data are plotted on the coordinates ϕ and $1/\psi$ as was done by Brown (18), this region is spread out and the data are seen to deviate sharply from both curves 1 and 2 of Fig. 6. This region of high values of τ_0 represents many of our streams and is of primary importance.

Kalinske (19) developed a theory for bed load transportation in which he expressed the rate of transport q_s of a sediment of uniform grain size by the equation,

$$q_s = \frac{\pi}{6} D^3 \rho_1 g \bar{U} g \frac{p}{\frac{\pi D^2}{4}} = \frac{2}{3} p d \rho_1 g \bar{U} g \quad (11)$$

where $\frac{p}{\frac{\pi D^2}{4}}$ is the number of grains per unit area of

bed, and \bar{U}_g is the mean velocity of the grains.

The instantaneous velocity U_g of a grain is taken as

$$U_g = b (u - u_c) \quad (12)$$

where b is a constant in the neighborhood of unity and u and u_c are respectively the instantaneous fluctuating velocity of the fluid in the vicinity of the grain and the fluid velocity that will set the grain into motion. Assuming that the fluctuation $(u - \bar{u})$ is normally distributed, the average value of $(u - u_c)$ can be calculated for $u > u_c$ with the result,

$$\bar{U}_g = \bar{U} f\left(\frac{u_c}{\bar{u}}\right) \quad (13)$$

Introducing Eq. 13 into Eq. 11 and noting that \bar{u} and u_c can be taken as proportional to the square root of the bed shears τ_o and τ_c , respectively, the bed-load equation can be written in the form

$$\frac{q_s}{\sqrt{\frac{\tau_o}{\rho}} \rho_l g D} = f_1 \left(\frac{\tau_c}{\tau_o} \right) \quad (14)$$

The functions f and f_1 also contain the turbulence intensity $\sqrt{(u - \bar{u})^2 / \bar{u}}$ but it does not appear explicitly in terms of τ_o or \bar{u} . In his analysis Kalinske used Eq. 3 to express τ_c .

Results of bed-load experiments are shown in Fig. 7 plotted by Kalinske according to Eq. 14. The data plotted on the figure are mostly for uniform size materials, but they also contain measurements by Einstein (17) on two small streams where the sediment is well graded. In Fig. 7, as in Fig. 6, the ordinate is proportional to the reciprocal of τ_o and

the data for high rates of transportation are compressed into a small region near $\tau_c/\tau_0 = 0$. In this region a given deviation from the curve represents a larger error than the same deviation for a higher value of τ_c/τ_0 . Therefore, it will be seen from Fig. 7 that the errors are higher at low values of τ_c/τ_0 where the transport rates are high.

Bed load formulas such as those of Einstein, Kalinske, and DuBoys depend on laboratory data for the determination of the functions or constants. The equations fit data for sands of uniform grain size moderately well. But in the much more complicated case where the grain sizes vary over a considerable range the correlation is much less satisfactory. In this case, as in natural streams, sorting of material in the bed no doubt is an important factor which contributes to the complexity of the problem. The formation of dunes on the bed modifies the roughness and in turn must reflect back on the transportation rate. When one attempts to apply these relations to streams, further complications are introduced. The stream is not straight like the flume, the cross section varies in shape and size, and the sediment has a large size range. These factors are sure to add to the deviations from the formulas already noted in the flume studies and thus increase the uncertainty of the result. This difficulty could be lessened if it were possible to measure the bed load of large streams and thus check the theories. In the absence of this possibility attempts are being made to refine the theories. Einstein (20) showed by analysis of laboratory results that the roughness of the bed increased as the bed load increased and that it correlated well with the transportation rate. Einstein and Barbarossa (21) also showed that the roughness of streams increased with the parameter ψ and ascribed this increase to bed irregularities or dunes. These resistance data are used by Einstein in the calculation of the flow depth to be used in his bed load relation.

The highest rates of transportation considered in the Einstein and Kalinske curves of Figs. 6 and 7

show the poorest correlation. Since this is just the range in which many of our rivers lie, it is of interest to improve our understanding of it. At these high rates of transportation considerable material is thrown into suspension and it appears that a formula must consider this fact if it is to predict the load.

Suspension of Sediment

The equation given by Schmidt (22) and O'Brien (23) for the distribution of C , the concentration of sediment grains with settling velocity w , in still fluid is, for a two-dimensional steady flow,

$$wC + \epsilon_s \frac{dC}{dy} = 0 \quad (15)$$

where y is the vertical distance from the bed and ϵ_s is a diffusion coefficient. The term on the right represents the upward rate of transport due to the vertical turbulent velocities and the term wC is the rate of settling due to the weight of the grains. If ϵ_s can be considered constant, Eq. 15 can be integrated to give the exponential relationship,

$$C/C_a = e^{-\frac{w}{\epsilon_s}(y - a)} \quad (16)$$

where C_a is the concentration at some arbitrary level $y = a$. Hurst (24) and Rouse (25) have checked this relation with observations of sediment concentrations in containers where water was agitated with paddles that gave essentially uniform turbulence throughout the liquid.

The coefficient ϵ_s in a turbulent stream is not constant but is an unknown function of y . Kalinske (26) has measured the diffusion coefficient in a water flow by applying the theory of Taylor (27), to analyze observation of the diffusion of droplets of a liquid mixture that had the same density as the water. Using these measured values of ϵ_s Kalinske

was able to predict observed distribution of sediment concentration in the same flow.

Another more convenient way to get ϵ_s is to assume as Rouse (28) did that ϵ_s is equal to ϵ_m the exchange coefficient for momentum, also sometimes called the eddy viscosity. The coefficient ϵ_m is defined by

$$\tau = \rho \epsilon_m \frac{du}{dy} \quad (17)$$

where τ is the shear at distance y from the bed. The quantity du/dy is obtained from the von Karman logarithmic velocity law,

$$\frac{u - u_{\max}}{U_*} = \frac{2.3}{k} \log \frac{y}{d} \quad (18)$$

where k is the von Karman universal constant with a value of 0.4 for clear fluids, d is the flow depth and $U_* = \sqrt{\tau_0/\rho}$, and is often called the shear velocity. Introducing du/dy from Eq. 18 into Eq. 17 and noting that in two-dimensional flow τ is a linear function of y , we obtain,

$$\epsilon_m = kU_* (1 - y/d) y \quad (19)$$

It is seen that the expression for ϵ_m is a parabola and that ϵ_m is zero at the bottom and surface of the stream and has its maximum value at mid-depth. Substituting Eq. 19 into 15 and integrating results in

$$\frac{c}{c_a} = \left[\frac{d - y}{y} \cdot \frac{a}{d - a} \right]^z \quad (20)$$

where

$$z = W/kU_* \quad (21)$$

The validity of the general form of this equation has been established by numerous laboratory (29) (30) and field (31) (32) measurements. These measurements show that the value of z which fits the data, referred to hereafter as z_1 , is usually smaller than

the value given by Eq. 21, indicating that the distribution of material is more nearly uniform than predicted by Eqs. 20 and 21. Figure 8 is a graph showing sediment distribution measurements along a vertical line in flows varying in depth from 0.3 ft to 10 ft. The solid lines follow Eq. 20 with values of the exponent indicated on the figure. In this figure the curves for z_1 values of 0.16, 0.43 and

1.12 are for the Missouri River at Omaha and are included here with the permission of the U. S. Engineer Dept. The data for the other four curves were obtained in a laboratory flume (33). It is seen that the theoretical curves fit the data very well. The fact that the theoretical exponent z does not agree with the observed exponent z_1 is not surprising since

it must be remembered that in developing the theory it was assumed that $\epsilon_s = \epsilon_m$ which is not necessarily to be expected. Since by correcting the exponent, Eq. 20 will fit the data, one can conclude that ϵ_s is proportional to ϵ_m or $\epsilon_s = \beta \epsilon_m$ which is equivalent to $z = \beta z_1$. The range of values of β obtained experimentally varies from about 1.0 to 1.5, indicating that ϵ_s tends to be larger than ϵ_m .

Equation 15 is derived on purely kinematic considerations and therefore does not account for the greater inertia of the sediment over that of the water. In this connection the experiments of Ismail (30) with 0.10 mm and 0.16 mm sands gave values of β of 1.5 and 1.3, respectively, thus indicating that the transfer coefficient diminishes as the sediment size increases. Laboratory experiments by the author (29) with the same sands used by Ismail, but in a larger channel, gave the same trend in β but gave different values. Rouse (25) also found in his

experiments with artificial uniform turbulence that the transfer coefficient was changing progressively as sediment increased in size from $1/32$ to $1/4$ mm. and argued that the effect was due to the greater inertia of the coarser particles. On the other hand, measurements in the Missouri River by the U. S. Engineer Department showed that at a given vertical section at a given time the value of β did not vary with sediment size. The decrease in β as the size of sediment increases is very likely the result of the increase in the slip between the fluid and grain as the particles get coarser. Since this is a dynamic effect, it is not described in Eq. 15.

Effect of Suspended Sediment on Flow Characteristics

Flume studies have shown that material in suspension has a pronounced effect on the characteristics of flow. The first effect that is noted when sediment is added to a flow is that the von Karman constant k , is reduced. Experiments (33) were made in which the depth and slope, and hence the shear, were kept constant while the suspended load was increased by adding sediment to the flow system. From Eq. 18 it is seen that since τ_0 , and hence U_* , does not change in the experiments, a decrease in k will cause an increase in the velocity gradient du/dy . Also from Eq. 19 it is seen that ϵ_m will decrease as k decreases, and from Eq. 17 we see that when τ remains constant a decrease in ϵ_m must be accompanied by an increase in du/dy . The effect of increasing the suspended load was to further decrease k . Fig. 9 shows values of k plotted against mean concentration of suspended load for three sets of measurements in which U_* is kept constant while the mean concentration varies. The data have considerable scatter but there is no doubt of the tendency for k to decrease as concentration increases. Values of k as low as 0.2 have been obtained for high concentrations. An increase in du/dy means that the mean velocity will

tend to increase, which means that the friction factor for the channel will decrease. Some experiments (33) showed marked increase in velocity and decrease in friction factor, while others showed little change. In the latter cases bed ripples are believed to have formed, thus increasing the resistance of the bed and compensating for the opposite effect of the increase in du/dy .

The decrease in k , and hence in ϵ_m , is visualized as resulting from the damping of the turbulence by the suspended material. The energy per unit volume and time, required to keep material from settling is $C w (1 - \rho/\rho_1)$. This energy which is provided by the turbulence will diminish the intensity of turbulence, and hence ϵ_m , since ϵ_m depends on the turbulence.

The details of this process are not clear. For instance, the energy per unit time required to support the sediment in the experiments of Fig. 9, is 3 per cent or less of the energy to overcome channel friction, and it is difficult to see how such small energy could produce such a large effect.

The exchange coefficient ϵ_s for sediment can be found from Eq. 15 and Eq. 20, with z replaced by z_1 . This gives

$$\epsilon_s = \frac{WY}{z_1} (1 - y/d) \quad (22)$$

and shows that ϵ_s is inversely proportional to z_1 .

Fig. 10 shows z_1 plotted against mean concentration (33) for experiments in which the data of Fig. 9 were obtained. z_1 is seen to increase with concentration, which means that ϵ_s decreases with increasing concentration and follows the trend of ϵ_m .

Laursen (34) has pointed out that the settling velocity is reduced as the sediment concentration increases due to interference between the flow fields of the grains. He presented results of McNown and Lin (35) giving correction factor for the settling velocity as a function of concentration for various Reynolds numbers. For a 0.1 mm. sand and a

concentration of 3 per cent by weight the reduction in settling velocity is about 20 per cent. The correction in the settling velocity has usually been neglected in calculations of sediment suspension.

Rate of Transportation of Suspended Load

The concentration, C_a , in Eq. 20 is not given by the theory of sediment suspension, and without it absolute values of the concentration cannot be calculated. This concentration in effect forms the connecting link between the bed load and suspended load and shows that the two modes of transportation are intimately related.

Lane and Kalinske (36) developed a relation for C_a in terms of the mechanical composition of the sand on the bed and the vertical component of turbulence fluctuations near the bed. They assumed that a vertical turbulence velocity that is greater than the settling velocity of a particle will place it in suspension. They also assumed that the rate of pick-up of particles of a given size from the bed is proportional to: (1) the relative amount of these particles in the bed, (2) to the magnitude of the vertical velocity capable of picking them up, and (3) to the relative amount of time during which velocities capable of picking up particles of this size exist. The pick-up rate is then proportional to

$$\Delta F(w) \int_w^{\infty} v f(v) dv \quad (23)$$

where $\Delta F(w)$ is the fraction of the bed sediment made up of grains with a settling velocity w , v is the vertical turbulent velocity, and $f(v)$ is the frequency function so that $f(v) dv$ is the fraction of the time that v has values between v and $v + dv$. In the development $f(v)$ was assumed to be the normal error function. Introducing the relation that the pick-up must equal the rate of settling cw and the

distribution function $f(v)$, we get the Lane-Kalinske relation,

$$C w \alpha = \Delta F(w) U_* \int_{t_c}^{\infty} t e^{-t^2} dt \quad (24)$$

where $t = v/U_*$ and $t_c = w/U_*$.

Fig. 11 shows data from large natural streams plotted to the coordinates t_c and $C/\Delta F(w)$ with C expressed in parts per million and $\Delta F(w)$ in per cent. The curve in the figure is of the equation,

$$C/\Delta F(w) = 5.55 p^{1.61} \quad (25)$$

$$\text{where } p = \frac{1}{t_c} \int_{t_c}^{\infty} t e^{-t^2} dt \quad (26)$$

This curve indicates that when t_c is in the neighborhood of unity very little material will be found in suspension. The above development was not carried to the point where C was associated with some elevation y so it could be used to get a value of C_a in Eq. 20.

Einstein (37) developed a method of calculating C_a using his $\phi-\psi$ relation of Fig. 6. He assumed that the $\phi-\psi$ relation will give the rate of sediment transportation q_B in a layer of thickness $2D$ directly above the bed. Then he takes the concentration C_{2D} of grains of a given size within this layer to be proportional to

$$C_{2D} \sim \frac{q_B i_B}{2D u_B} \quad (27)$$

where u_B is the flow velocity at the bed and i_B is

the fraction of the total bed material made up of the given size. Taking $u_B \sim U_*$ he finally gets

$$C_a = \frac{q_B^i}{2DU_* \cdot 11.6} \quad (28)$$

where the number $1/11.6$ is the experimentally-determined constant of proportionality. This concentration C_a is assumed to occur at a distance $y = 2D$ above the bed and is used in Eq. 20 to calculate the suspended sediment.

Einstein and Barbarossa (21) explained the observed variation of the roughness of natural streams with flow rate by postulating that the total resistance to flow was made up of two parts: (1) a part resulting from the constant and predictable roughness of the sand grains, and (2) a part resulting from the variable roughness of the bed irregularities. These authors also expressed the resistances or bed shears τ'_0 and τ''_0 due to sand grains and dunes, respectively, in terms of fictitious hydraulic radii R' and R'' defined by,

$$\tau'_0 = \rho g R' S \quad (28)$$

$$\tau''_0 = \rho g R'' S \quad (29)$$

where S is the slope of the stream. In calculating ψ to use in Fig. 6 to obtain ϕ and hence, q_b for use in Eq. 28, the fictitious value R' from Eq. 28 is used instead of the real value R of the hydraulic radius.

Einstein (37) calculated the rate of transportation of suspended load q_s from the equation

$$q_s = \int_{2D}^d u C dy \quad (30)$$

using Eq. 20 to express C , Eq. 28 to express C_a , and the von Karman-Nikuradse logarithmic equations to express u . These relationships have been checked against results of sediment transportation experiments in a flume, but stream data for checking them are not available. A number of the essential factors such as the exponent z and the von Karman constant k , vary strongly with the load and cannot be estimated precisely. An error of as little as 20 per cent in z when z is in the neighborhood of 1.0 will produce an error in the calculated rate of transportation of as much as 100 per cent. This fact emphasizes the need for obtaining more reliable information on the effects of the sediment on these factors.

Summary

Even a brief survey of the mechanics of sediment transportation, such as the present one, indicates clearly that sediment movement is intimately associated with turbulence. Observations of the erratic intermittent motions of grains on a bed are explained with any degree of satisfaction only in terms of the pattern of turbulence in the flow, and the behavior of suspended material is even more closely tied in with turbulent motion. The theories presented in this summary represent the best information that is available at this time, yet it is clear that they fall short of achieving a quantitative description of the phenomena.

In the early days of sediment transportation investigations, attempts were made to develop simple transport formulas that could be applied directly to engineering problems. This ambitious objective met with little success, and researches in recent years have been going more and more into the study of the basic phenomena involved and have clarified some of the problems. Much yet remains to be done on this important and interesting but very complicated problem.

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Fig. 1 - Sand streaks blowing across road in wind storm.

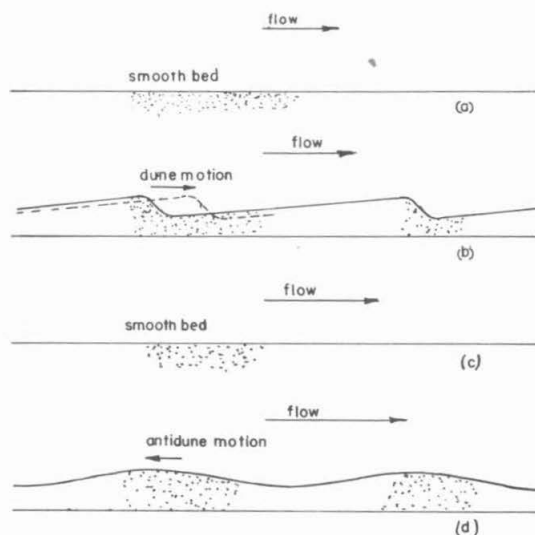


Fig. 2 - Diagrams of a stream bed showing four bed configurations at progressively increasing flows.

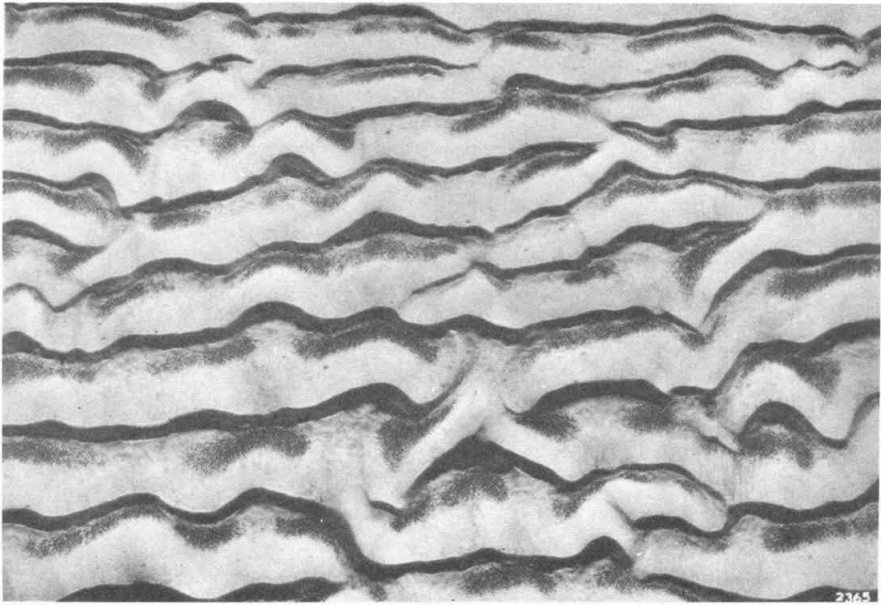


Fig. 3 - Dunes Formed on the Bed of a Flume by a Flow 0.295 Deep on a Slope of 0.00125

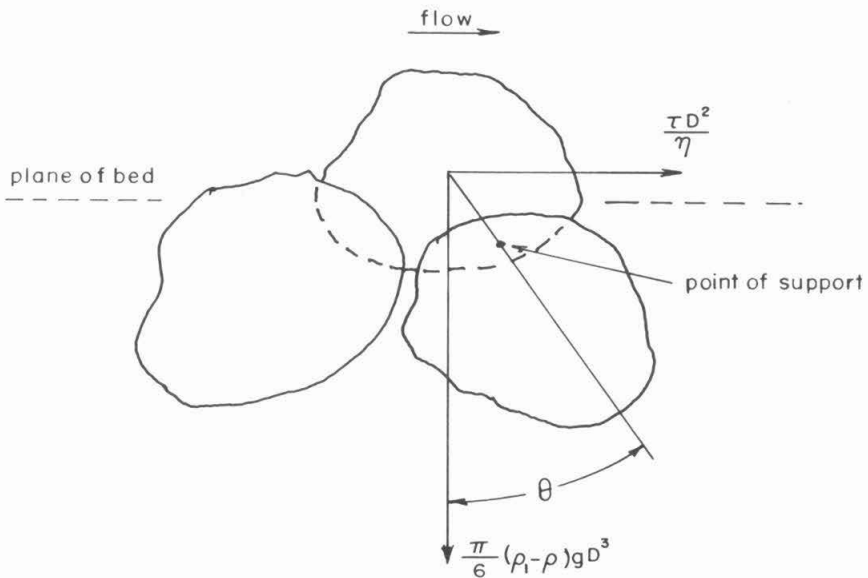


Fig. 4 - Diagram Showing Forces on Sand Grain in the Bed of a Stream

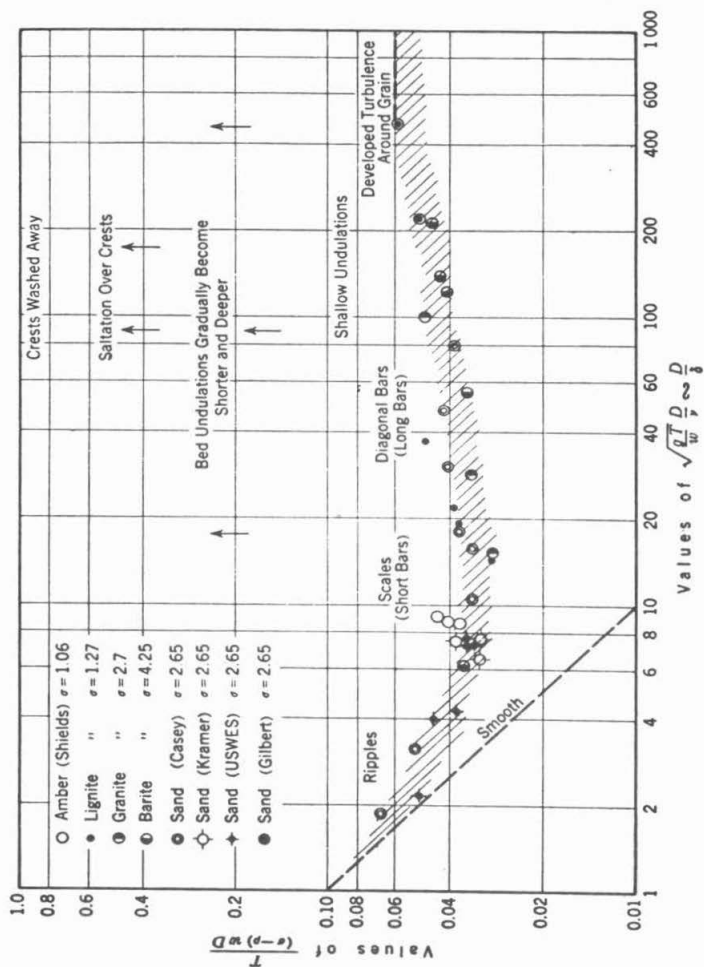
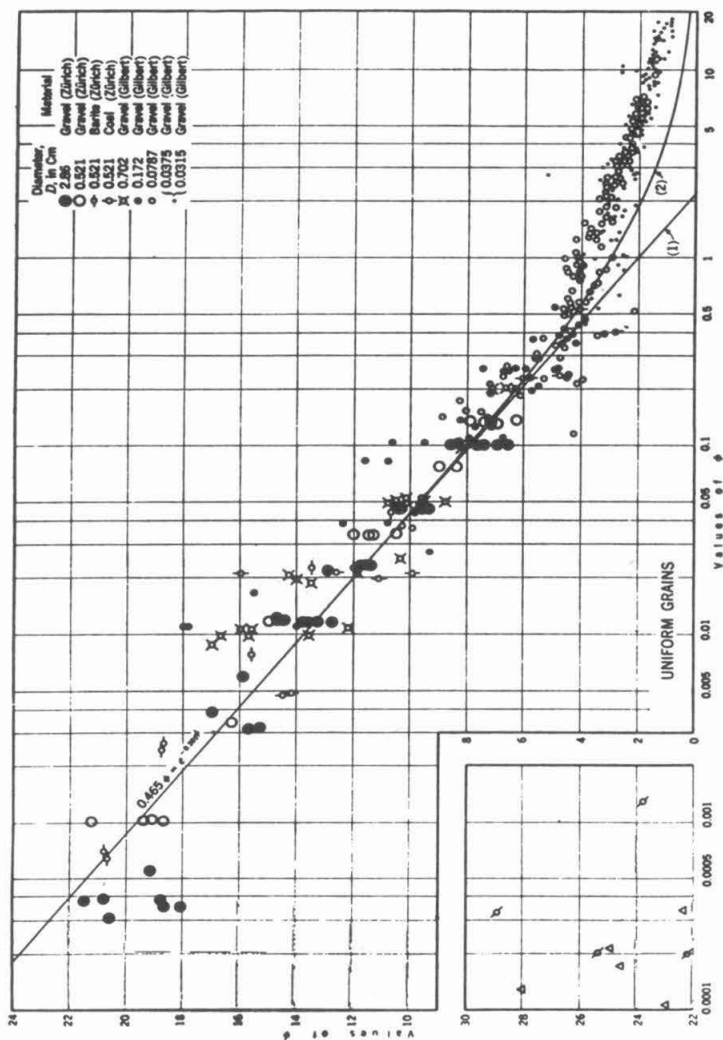


Fig. 5 - Critical tractive force plotted against Reynolds number of sand grain according to Shields.

Fig. 6 - Graph of the Einstein $\phi - \psi$ bed load function.

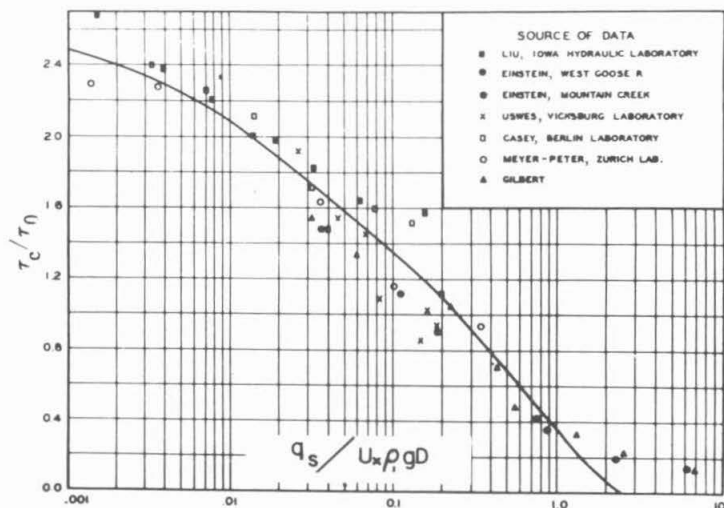


Fig. 7 - Graph of the Kalinski bed load equation.

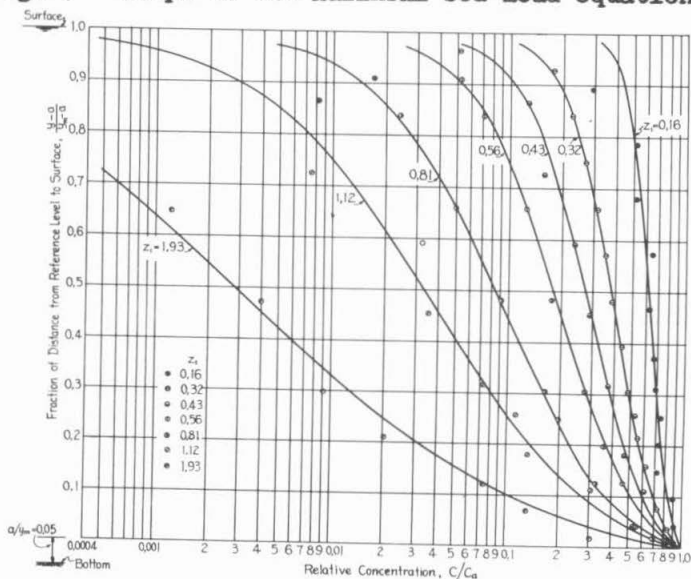


Fig. 8 - Comparison of suspended sediment distribution measurements in flumes and rivers with the theoretical distribution curves.

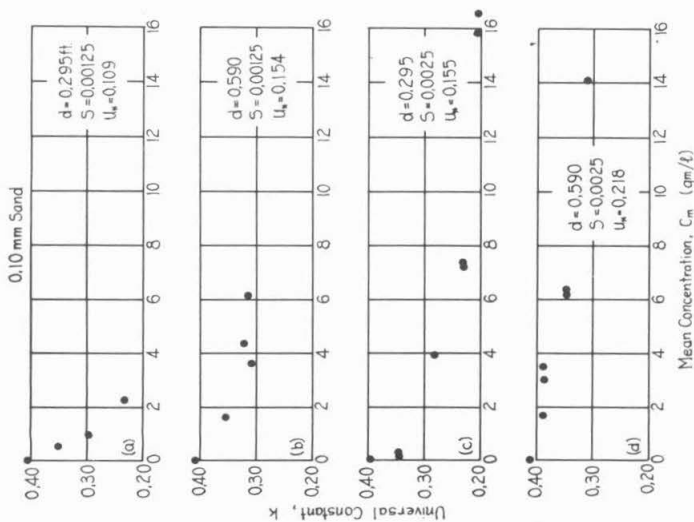


Fig. 9 - Variation of the von Karman Factor k with Concentration of Suspended Sediment

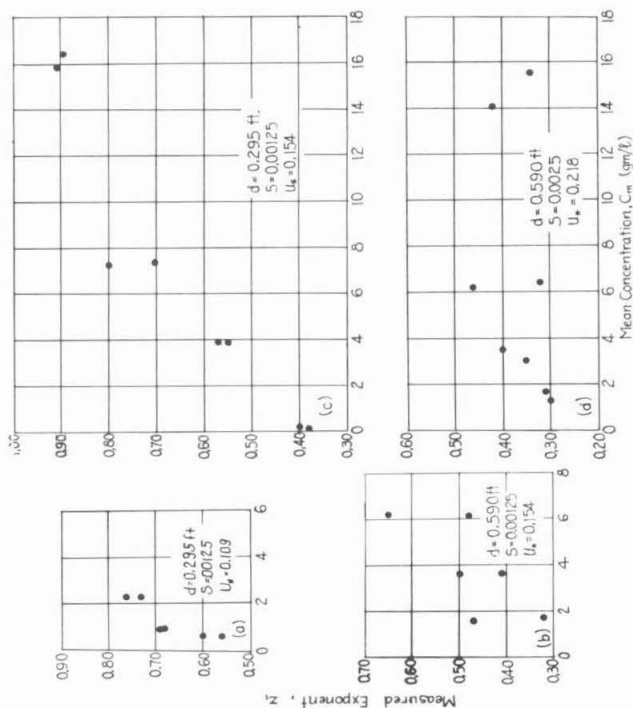


Fig. 10 - Variation of the Exponent Z_l in Eq. (20) with Concentration of Suspended Load

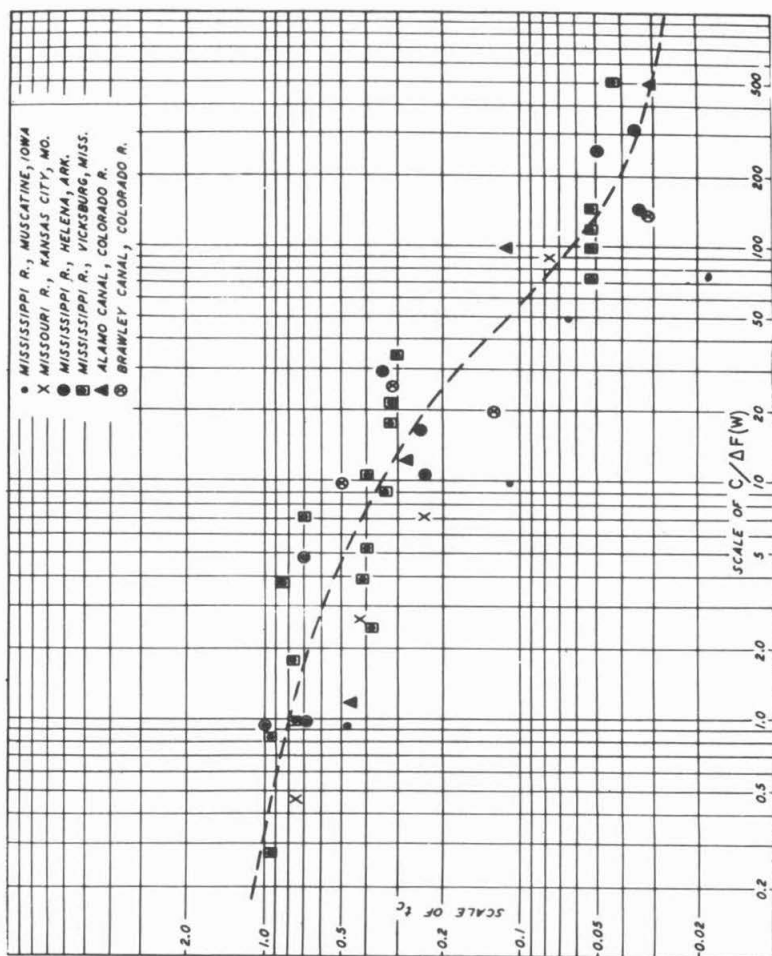


Fig. 11 - Kalinski relation between suspended and bed material.

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